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# SLUG FLOW BEHAVIOR IN A HILLY TERRAIN PIPELINE

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**Abstract**—When slugs flow in a hilly terrain pipeline that contains sections of different inclination they undergo a change of length as the slugs move from section to section. In addition, slugs can be generated at low elbows, dissipate at top elbows and shrink or grow in length as they travel along the pipe. In this work a slug-tracking model is proposed that follows the behavior of all individual slugs and is capable of simulating the aforementioned processes. Two cases are considered: the case of steady slug flow, for which each slug maintains its identity as it flows from one section to another; and the more complex case, where new slugs are generated and disappear, and the slug identity along the hilly terrain is not maintained. Comparisons with experimental data demonstrate the capability of this slug-tracking method and show that the proposed model is able to simulate correctly slug behavior in a hilly terrain pipeline.

*Key Words:* slug flow, slug tracking, slug length, hilly terrain pipeline.

## INTRODUCTION

Slug flow is one of the most common flow patterns in two-phase pipe flow. Modeling slug flow for the purpose of calculating slug flow parameters was first proposed by Dukler & Hubbard (1975). The general approach of Dukler & Hubbard (1975) was later used and/or modified by Nicholson *et al.* (1978), Stanislav *et al.* (1986) and Taitel & Barnea (1990a). Similar models were also proposed for vertical flow by Fernandes *et al.* (1983), Orell & Rembrand (1986), Sylvester (1987) and Barnea (1990). Critical review papers on slug flow can be found in Taitel & Barnea (1990b) and Fabre & Liné (1992).

In all the models proposed, one considers an average slug unit and the flow is assumed to consist of equal-length slugs. In spite of the relative progress in slug flow modeling, the slug length is still a variable that is difficult to predict. Also, the assumption of equal length is obviously not accurate and there is considerable statistical variation of slug lengths in the pipe.

The process of slug growth is controlled by the pick-up process at the slug front, where the film ahead of the slug is scooped by the approaching slug front, and by the shedding at the back of the slug. The shedding process is determined by the translational velocity of the bubble that penetrates into the rear of the liquid slug. This translational velocity depends on the velocity profile of the liquid at the rear of the slug and is approximately equal to the maximum local velocity. For fully developed slug flow this maximum velocity is about $1.2U_s$ (for turbulent flow). For steady stable slug flow, the translational velocity of all bubbles is the same. As a result, the velocities of the front and back of the slugs are equal, and the slug lengths remain constant. However, for short slugs, the velocity profiles at the rear of the slugs are not yet fully developed and the maximum velocity is $> 1.2U_s$. As a result, short slugs behind long stable slugs tend to disappear as the trailing translational velocity at the rear of the slug overtakes the slug front (Moissis & Griffith 1962; Taitel & Barnea 1990b). Therefore, short slugs are unstable and tend to merge with the longer slugs behind them. Taitel *et al.* (1980) and Barnea & Brauner (1985) suggested that a fully developed slug length is equal to the distance at which a jet is being absorbed by the liquid. Using this approach, an estimated value of $16D$ ($D$ is the pipe diameter) was obtained for the minimum liquid slug length in vertical upflow and a value of $32D$ was obtained for the case of horizontal flow.

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Slugs are generated at the entrance section of the pipe. Usually, however, the frequency of generation of slugs is relatively high. As a result, a series of short slugs are generated which tend to merge and form longer slugs. This merging process continues until the liquid slugs are long enough to be stable, namely the trailing bubble is unaffected by the wake of the leading one. This occurs when the velocity profile at the rear of the liquid slug can be considered fully developed (Taitel *et al.* 1980; Barnea & Brauner 1985; Dukler *et al.* 1985).

Another problem also related to the slug length is the change of slug length as it travels in a hilly terrain pipeline with upward and downward sections. Slug length tends to increase as it moves from a horizontal or downward section into an uphill section, as a result of liquid accumulating in the low elbow and being picked up by the approaching slugs (Scott & Kouba 1990). In addition, new slugs can be generated in low elbows. These slugs usually are short and therefore unstable and tend to dissipate. In this work, a model is proposed that is capable of tracking individual slugs and simulating the basic mechanisms of slug growth, generation and dissipation that take place when slugs travel in a pipe which changes its inclination.

## STEADY SLUG FLOW

The simplest case of a change in slug length is when the slugs are sufficiently long and close to each other that each slug maintains its identity when the pipe inclination is changed. The average flow rate of liquid at any cross section of the pipe is

$$W_L = \frac{1}{t_u}\left(U_L A R_s \rho_L t_s + \int_0^{t_f} U_f A R_f \rho_L \, dt\right), \qquad [1]$$

where $W_L$ is the input liquid flow rate, $U_L$ is the average liquid velocity in the slug, $U_f$ is the liquid velocity in the film, $A$ is the pipe cross-sectional area, $\rho_L$ is the liquid density and $R_s$ and $R_f$ are the liquid holdups in the slug and the film zones, respectively, and $t_u$, $t_s$ and $t_f$ are the times for the passage of the slug unit, the liquid slug and the film zone, respectively. Since $t_s = l_s/V_t$ and $t_f = l_f/V_t$, [1] takes the form

$$W_L = U_L A R_s \rho_L \frac{l_s}{l_u} + \frac{1}{l_u}\int_0^{l_f} U_f A R_f \rho_L \, dX, \qquad [2]$$

where $l_s$, $l_f$ and $l_u$ are the liquid slug, film zone and slug unit lengths, respectively, and $V_t$ is the slug translational velocity.

Simplifying the slug structure as consisting of a film of constant thickness, then the continuity of liquid flow rate can be written as

$$W_L = U_L A R_s \rho_L \frac{l_s}{l_u} + U_f A R_f \rho_L \frac{l_u - l_s}{l_u}. \qquad [3]$$

A mass balance of liquid relative to the translational velocity yields

$$(V_t - U_f)R_f = (V_t - U_L)R_s. \qquad [4]$$

For steady-state flow the liquid flow rate in each section of the pipe is the same for any angle of inclination. Also, the frequency of the slugs given by

$$\nu = \frac{V_t}{l_u} \qquad [5]$$

is constant.

Considering a flow from section I to section II for which the pipe diameter and liquid density are the same (see figure 1), equating the liquid flow rate [3] in both sections and using [4] and [5], yields the following relation for the slug length ratio of sections I and II:

$$\frac{l_{sII}}{l_{sI}} = \frac{R_{sI} - R_{fI}}{R_{sII} - R_{fII}} + \frac{U_{fI} R_{fI} - U_{fII} R_{fII}}{V_{tI}(R_{sII} - R_{fII})} \frac{l_{uI}}{l_{sI}}. \qquad [6]$$

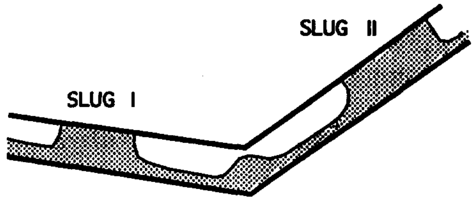



Figure 1. Slug growth in an elbow.

For the special case when the translational velocities for both sections are the same (this is usually the case for small changes in the inclination angle), then

$$(V_{tI} - U_{fI})R_{fI} = (V_{tII} - U_{fII})R_{fII} \qquad [7]$$

and [6] takes a simpler form:

$$\frac{l_{sII}}{l_{sI}} = \frac{R_{sI} - R_{fI}}{R_{sII} - R_{fII}} + \frac{R_{fI} - R_{fII}}{R_{sII} - R_{fII}} \frac{l_{uI}}{l_{sI}}. \qquad [8]$$

Equation [8] shows that when considering a bottom elbow, i.e. when the angle of the pipe changes from downward or horizontal to upward, the length of the slug increases. Since film holdup for upward flow is always less than the film holdup for downward flow ($R_{fI} > R_{fII}$), then $l_{sII}$ is always larger than $l_{sI}$. Equation [8] also shows that the slug length ratio increases when the slug unit is larger ($l_{uI}/l_{sI}$). This is quite obvious on physical grounds. The low elbow accumulates liquid from the incoming pipe I (if it has a negative slope) as well as from the outgoing pipe II. This accumulated liquid is picked up by the liquid slug just before it enters the upward-sloping pipe. As a result, the liquid slugs in the uphill section are longer than those in the downhill section. The longer the slug unit length $l_u$, the more liquid is picked up and the longer is the slug in the uphill section.

For a top elbow the inverse is the case. The slug length is shorter in the downhill section. Note that for the downhill section the slug length may shrink to zero, in which case there is no slug flow solution for the downhill section of a top elbow. This condition occurs (using [8]) when

$$\frac{l_{uI}}{l_{sI}} \geqslant \frac{R_{sI} - R_{fI}}{R_{fII} - R_{fI}}. \qquad [9]$$

This phenomenon is also obvious on physical grounds. When a liquid film is at the top elbow, it is being drained in both directions, leaving the top elbow dry. A liquid slug that passes through the top dry elbow does not pick up any liquid at the front, but it sheds liquid from the back. As a result, the liquid slug length decreases when passing through a top elbow. For cases where [9] is satisfied, the liquid slug will dissipate completely.

## SLUG GENERATION AT THE BOTTOM ELBOW

Figure 1 shows the process by which new slugs may be generated at the bottom elbow. Liquid flows into the bottom elbow from both sections I and II and accumulates at the elbow. If the accumulated liquid is sufficient to block the gas passage, then a new slug will be formed and will travel into section II. Otherwise, the liquid will be picked up by the next liquid slug that enters section II. The latter process maintains the slug identity as it travels from section I into section II, as described in the previous section.

The amount of liquid that has to be accumulated in order to generate a new slug is unknown. In this work, we will assume that this amount is known and is given in terms of a known slug length $l_{gen}$. When the accumulated liquid at the bottom elbow is sufficient to generate a liquid slug of length $l_{gen}$, before it is picked up by the preceding liquid slug, then a new slug is formed.

The accumulation rate of liquid at the bottom elbow in terms of the slug length in region I is given by

$$(U_f A R_f \rho_L)_I - (U_f A R_f \rho_L)_{II} = A(R_{sI} - R_{fI})\rho_L \frac{dl}{dt}. \qquad [10]$$

The LHS of [10] is the liquid flow rate into the elbow from the film in regions I and II. The liquid accumulated is being either picked up by the next incoming slug or it will form a new slug of length $l_{gen}$. It is assumed that the slug is generated at the elbow and is accumulated in the incoming pipe I. Therefore, the RHS of [10] is the liquid accumulation rate in terms of the hypothetical slug length, $l$, generated in region I. A new slug is formed once this length reaches the prescribed value of $l_{gen}$ before the next slug passes through the elbow.

## DISAPPEARANCE OF SLUGS

Slugs will maintain their length provided the shedding from the back and the pickup at the front is the same. Short slugs behind long slugs tend to be unstable, since the bubble front velocity behind the short slug travels faster than bubbles behind long liquid slugs where the profile is fully developed (Taitel & Barnea 1990b). The minimum length of stable slugs is of the order of 15–30$D$. In this work we will consider the minimum length of a stable slug as an input parameter, $l_{stab}$. The bubble velocity behind a stable liquid slug is correlated as

$$U_b = V_t = CU_s + U_d \simeq CU_s. \qquad [11]$$

Note that the bubble velocity, $U_b$, is essentially the translational velocity of the rear slug boundary, $V_t$. The drift velocity, $U_d$, is usually small. The bubble velocity behind a short slug is approximated by the following relation for slug length shorter than the stability length:

$$U_b = U_{bmax} - (U_{bmax} - CU_s)\frac{l_s}{l_{stab}}. \qquad [12]$$

In [12] the bubble velocity is linearly related to the length of the liquid slug. It is not claimed here that this is indeed the correct relation. This is just an example to demonstrate how short slugs behind long slugs behave as they travel downstream. For a real simulation, the correct dependence of $U_b$ on the slug length $l_s$ should be implemented. This is, however, outside the scope of this work.

## SLUG-TRACKING SIMULATION

Based on the abovementioned concepts, it is possible to simulate slug movement, growth, generation and disappearance by tracking each individual slug. This capability is demonstrated for the case shown in figure 2, namely when horizontal slug flow enters an uphill section, followed by a downhill section and then another horizontal section. The first elbow is at $X = 0$; the second one, at the top, is at $X = X_{TOP} = 60$ m; and the third elbow is at $X = X_{BOTTOM} = 120$ m. Each slug is sequentially numbered by an index $i$. The position of each slug is determined by 4 variables: $x_i$, $y_i$, $w_i$ and $z_i$; where $x_i$ is the $X$ position of the front of slug $i$ in the pipe and $y_i$ is the position of the back of slug $i$ (see figure 2). $w_i$ and $z_i$ are used only when the film is at an elbow and splits

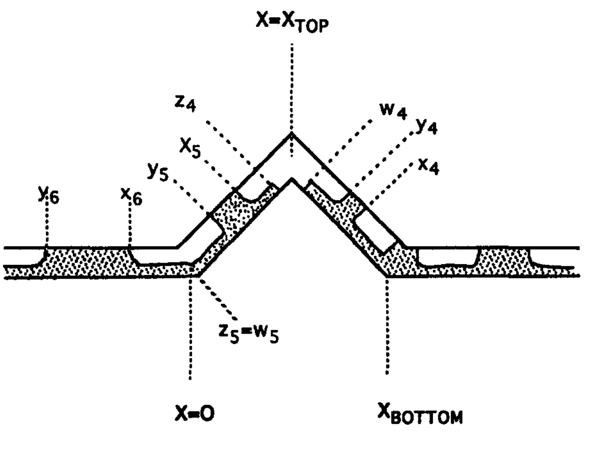



Figure 2. Geometry of the pipeline and the coordinate system.

into two parts, otherwise $w_i = z_i = y_i$. In a top elbow, the downstream part in the downhill section is located from $y_i$ to $w_i$ and the upstream part is located from $z_i$ to $x_{i+1}$ (see figure 2 for $i = 4$). For a low elbow, $w_i = z_i = 0$ (see figure 2 for $i = 5$). The simulation starts with a known initial condition. The positions of all the slugs are specified by the $x_i$, $y_i$, $w_i$ and $z_i$ values. The properties for each slug are then calculated. The liquid holdup within the slug body, $R_{si}$, can be calculated from an appropriate model or correlation (Gregory *et al.* 1978; Barnea & Brauner 1985). In this simulation we used the Gregory *et al.* (1978) correlation, which was obtained for conditions similar to our experiments. Thus,

$$R_s = \frac{1}{1 + \left(\frac{U_s}{8.66}\right)^{1.39}}. \qquad [13]$$

The translational velocity $V_{ti}$ was calculated using [12], where $C$ was taken as 1.2 and $U_{bmax}$ as $1.06CU_s$ (an arbitrary value chosen for the convenience of presentation). The liquid slug mixture velocity $U_{si} = U_{LS} + U_{GS}$ is a known input variable. It can be a function of time; but in this simulation it is constant and equals 5 m/s. The film is considered to have a uniform thickness and velocity. We considered the film velocity, $U_{fi}$, to be known as a function of the inclination angle, $\beta$. The film holdup $R_{fi}$ can then be calculated by

$$R_{fi} = \frac{(V_{ti} - U_{si})R_{si}}{V_{ti} - U_{fi}}. \qquad [14]$$

The film velocity was taken here as 0 for the horizontal section, +1 m/s for the downward-inclined section and −1 m/s for the upward-inclined section. Note that in reality the situation is more complex, since the equilibrium film velocity depends also on the film holdup and the actual film holdup has to be calculated via the solution of [14] and the equilibrium momentum equation for the film flow. For the sake of simplicity we used a known film velocity in this simulation. Usually the film velocity is sensitive to the inclination but not too sensitive to the liquid holdup within a narrow range of the holdup.

Once the liquid holdup $R_{fi}$ is calculated, the front velocity of the liquid slug is calculated from a mass balance on the front slug boundary to yield:

$$V_{fi+1} = \frac{U_{si+1}R_{si+1} - U_{fi}R_{fi}}{R_{si+1} - R_{fi}}. \qquad [15]$$

After all the translational velocities $V_{ti}$ and the front velocity $V_{fi}$ are known, the movement of the front and back boundaries of each slug can be calculated during a time interval $\Delta t$. Thus, a valid tracking procedure is possible.

When a film is in an elbow, the film is split into two zones: the film to the "right" of the elbow and the film to the "left" of the elbow. The location of the split is given by the values $w_i$ and $z_i$. For the low elbow $w_i = z_i = 0$, while for a top elbow $w_i$ and $z_i$ are different, as shown in figure 2, due to the splitting of the film into two separate films. Note also the liquid holdups of the two film zones are different.

The mechanism of slug growth and generation at the low elbows ($X = 0$ and $X = X_{BOTTOM}$) (see figure 2) is modeled as follows. In a low elbow, liquid is accumulated at the rate

$$\frac{dv_0}{dt} = (U_f A R_f)_I - (U_f A R_f)_{II}. \qquad [16]$$

In [16], I refers to the film on the "left" (upstream) and II to the film on the "right" (downstream); $v_0$ is a hypothetical sink that accumulates the liquid at the low elbow. As time progresses, one of two events may take place: (1) the slug that approaches this elbow from the "left" will collect the liquid within the sink as it passes through the elbow; or (2) the liquid volume $v_0$ becomes sufficiently large that it blocks the air passage, resulting in the generation of a new slug at the elbow. As mentioned, the criterion for this generation is entered in terms of a generation length, $l_{gen}$, which must be an input parameter,

$$v_0 = l_{gen}(R_{sI} - R_{fI})A. \qquad [17]$$

Note again, that it is assumed that the slug formed in the elbow is generated in the upstream ("left") section.

The capability of this method for tracking slugs is demonstrated in figures 3–13. The simulation starts with the known initial positions of all the slugs present in the pipeline. For time $t = 0$ the position of each slug is designated by the positions of the front and back boundaries of each slug. In figure 3 we simulate the movement of the 34 slugs initially present in the pipe by tracking the $X$ positions of $x_i$, $y_i$, $w_i$ and $z_i$ with time. The initial length of each slug is $30D$ (1.5 m for $D = 0.05$ m) and the film length is $500D$ (25 m). Slug No. 2 is the first one (2 for programming convenience) and is located in the uphill section. Slugs Nos 3–34 are in the horizontal section at time $t = 0$. Owing to the small scale of figure 3, the distances between the front of the slugs and the back are almost indistinguishable. A more detailed picture is shown in figure 4 for the first 20 s. One can see clearly that slug No. 2 decreases in length because there is no film ahead of the slug to be picked up, and the front velocity $V_f$ equals the mixture velocity ($U_s = 5$ m/s), while the back of the slug moves at the translational velocity $V_t (= 6$ m/s). Once the slug body disappears ($x_2 = y_2$), and since the liquid film flows "backwards" in the uphill section, we can see that the film boundary $y_2$ moves backwards (the $X$ position of $y_2$ decreases) until it is overtaken by slug No. 3. Observing the behavior of slug No. 3 we can see that for $X < 0$ the slug length is 1.5 m. Once the slug passes the low elbow at $X = 0$, its length increases to about 2.2 m and then remains constant as long as the film ahead of slug No. 3 is present. Once the film ahead of slug No. 3 is overtaken by the slug, the length of the slug decreases and the slug disappears. Thus, as can be seen from figures 3 and 4, slugs Nos 2–7 disappear before reaching the top elbow. Slug No. 8 reaches the top of the elbow at $X = 60$ m. After passing the top elbow, the film behind this slug splits at the top into two sections. The detail is shown in figure 5. As slug No. 8 passes the top elbow, the two boundaries $w_8$ and $z_8$ that were equal to $y_8$ are now shown (see also figure 2). The film boundaries downhill ($X > 60$) travel forwards at the film velocity $U_f = +1$ m/s, while the film boundary in the uphill section travels backwards (as shown in figure 5). The region between the two boundaries $w_8$ and $z_8$ is the dry region at the top elbow. When slug No. 9 reaches the dry zone it decreases in size until it overtakes the film in the downhill section. At this point the length of slug No. 9 remains constant until it overtakes the

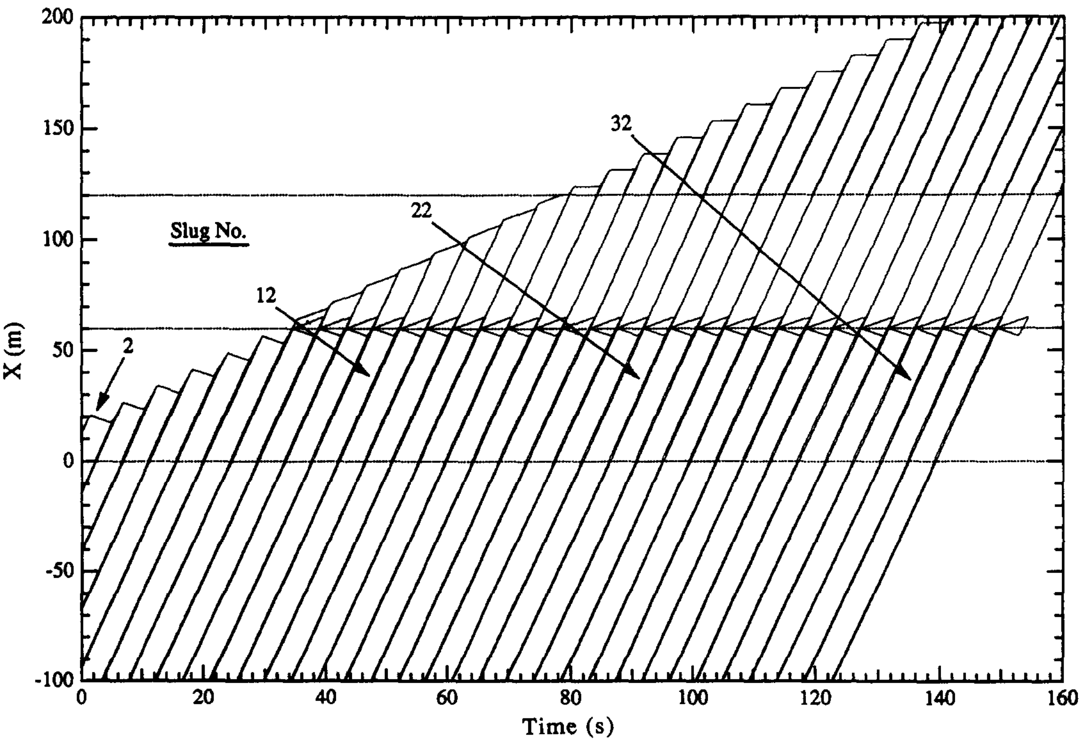



Figure 3. Slug tracking in hilly terrain geometry.

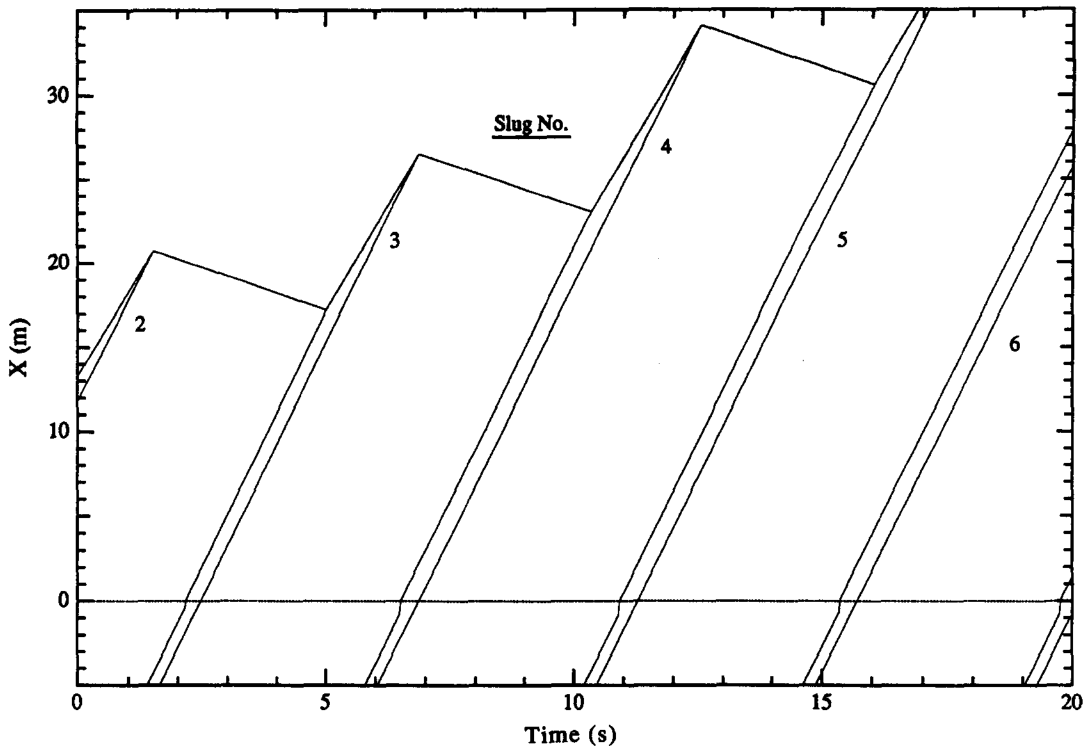



Figure 4. Close-up of figure 3 for the first 20 s.

front boundary of the film. In this process the slug length changes and becomes quite short ($\simeq 0.45$ m). Thereafter, the slug decreases in size until it disappears, since there is no film ahead of it. In this process slugs Nos 8–15 decrease in size and disappear before reaching the bottom elbow at $X = X_{\mathrm{BOTTOM}} = 120$ m. Slugs No. 16 and higher do reach the second bottom elbow and increase in length when reaching the bottom elbow, as described for the first elbow (at $X = 0$). It is interesting to observe that the original slug length of 1.5 m is regained when the slugs enter the horizontal section again. Obviously this is required on the basis of mass continuity. Note that for this simple case of steady flow, the slug length in the uphill and downhill sections can be calculated easily by [6] or [8].

Figure 6 shows the liquid holdup in the pipe as a function of time for different $X$ locations. The first location is in the upstream horizontal section ($X = -30$ m); the second is in the middle uphill section ($X = 30$ m); the third is in the middle downhill section ($X = 90$ m); and the fourth is, again, in the horizontal section past $X_{\mathrm{BOTTOM}}$ ($X = 150$ m). For this idealized picture the holdups in the liquid slug and the film are essentially constant, and this is clearly seen in the figure. At early times the pipe is dry for the slugs Nos 2–4 locations until liquid reaches these locations. It is interesting to observe that, for the uphill and the horizontal sections, the first liquid arriving at the location is in the slug body, while for the downhill section the film preceded the slug body. This is because the film flows "backwards" in the uphill section while the film velocity is zero in the horizontal section and is positive in the downhill section.

Figure 7 is a close-up of figure 6 for the time between 120 and 130 s. It is clearly seen that for the upward-inclined case, the time for the slug passage is longer than for the horizontal case. Since the translational velocity is constant, the longer time of passage means that the uphill slugs are longer. Likewise, we see that in the downhill section the slugs become quite short, but regain their original length in the horizontal section. This figure is particularly useful since, in an experiment, one usually measures the holdup at fixed locations and the type of data obtained is in this form.

Figure 8 shows the variation of the length of slug No. 32 with time. As seen, for early times the slug is in the horizontal section, far upstream, and its length is 1.5 m ($30D$). When it reaches the

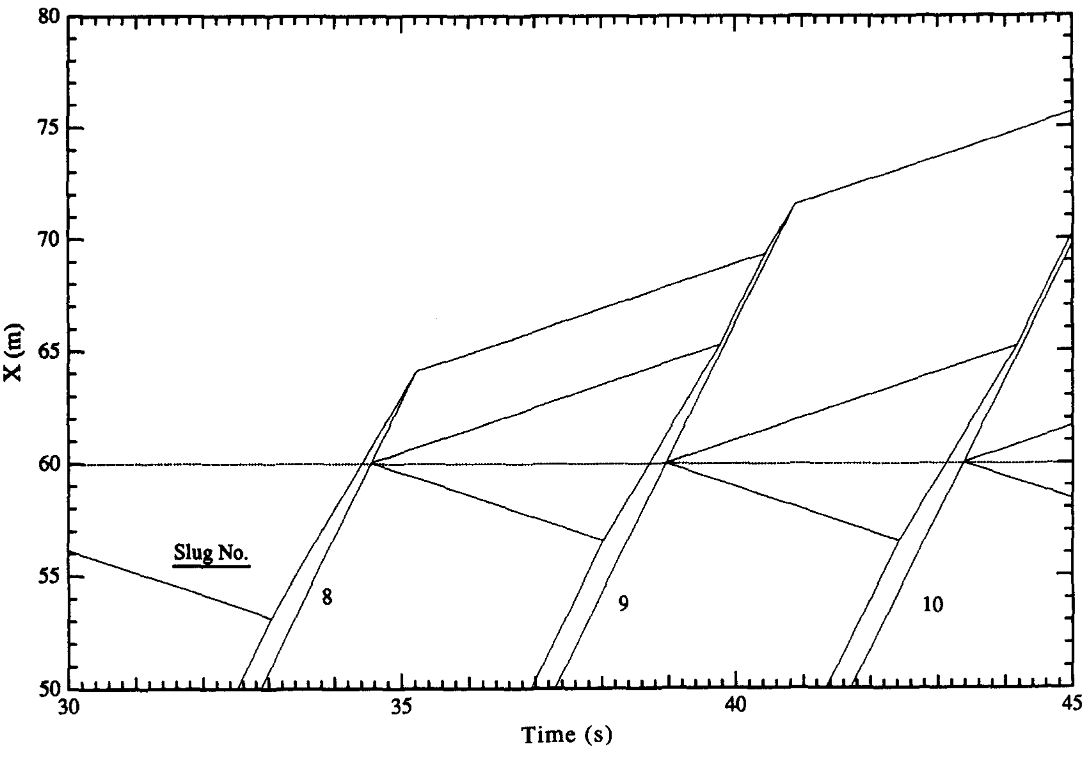



Figure 5. Close-up of figure 3 between 30 and 45 s.

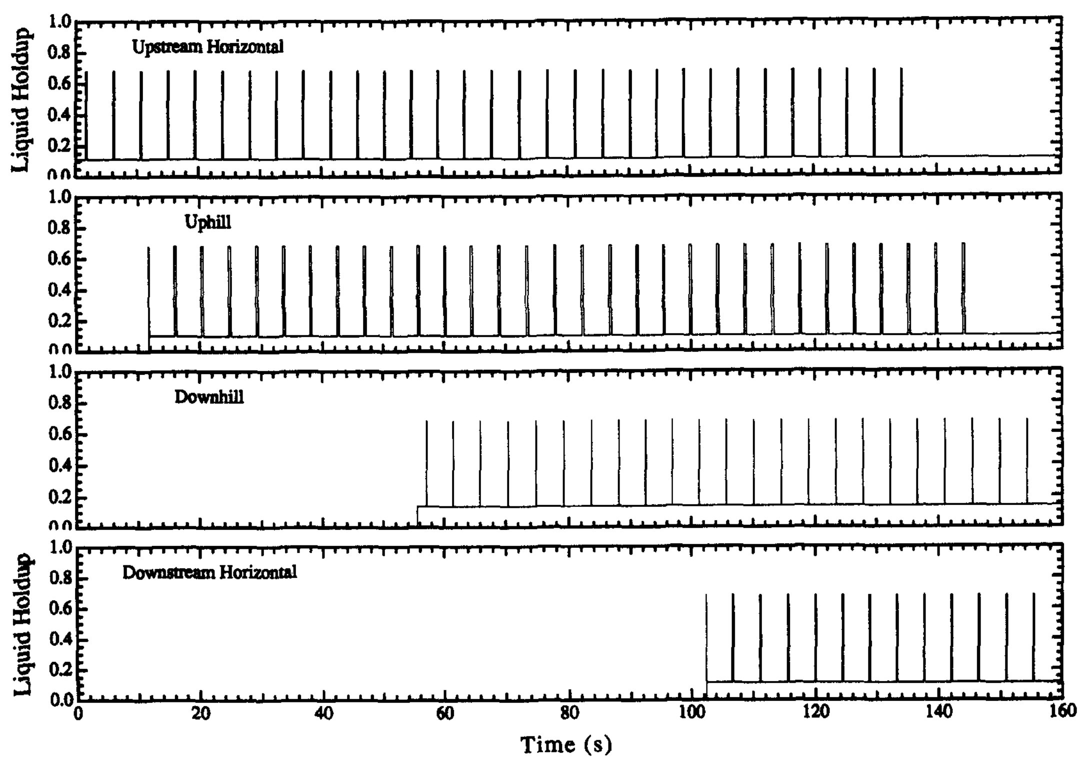



Figure 6. Liquid holdup as a function of time.

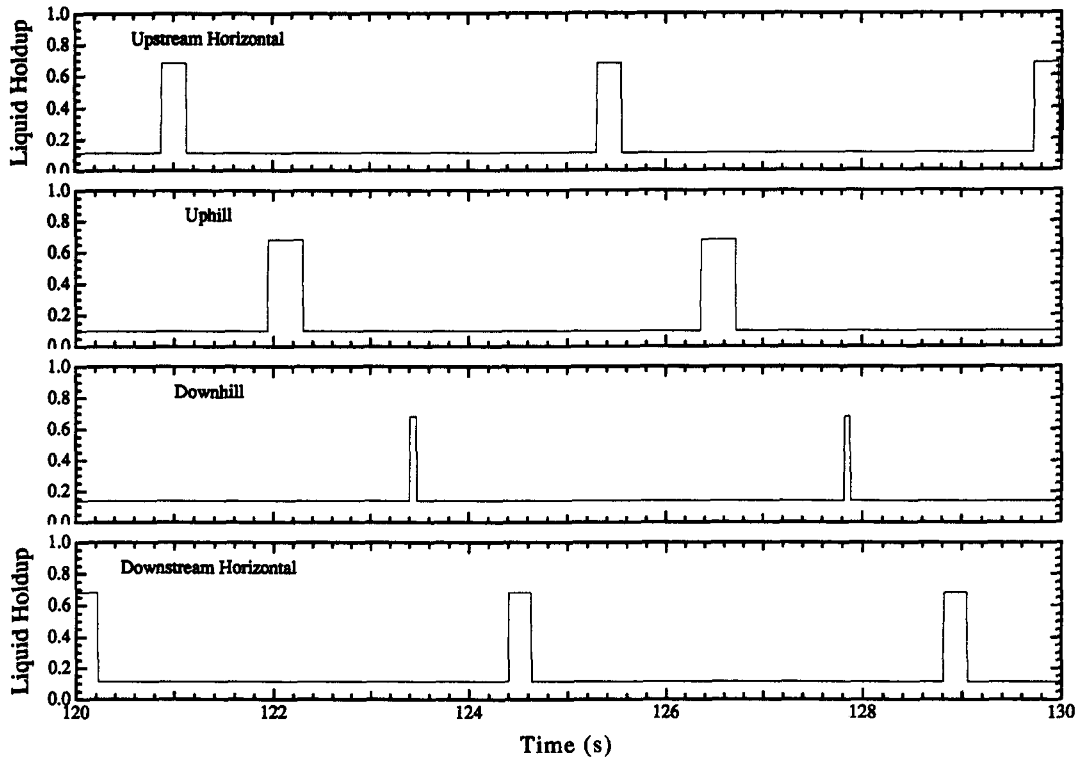



Figure 7. Close-up of figure 6 between 120 and 130 s.

uphill section the slug length increases to about 2.2 m. In the downhill section it decreases to about 0.45 m. Finally, the slug length returns to 1.5 m in the horizontal downstream section.

The examples reported in figures 3–8 are for the simple case where slugs maintain their identity. In the inclined sections the lengths of the slugs are changed but each slug keeps its identity along the pipe.

The case shown in figure 9 is when the film length between the slugs is increased from $500D$ to $800D$ (40 m). In this case the slug length in the uphill section is larger than before, but in the

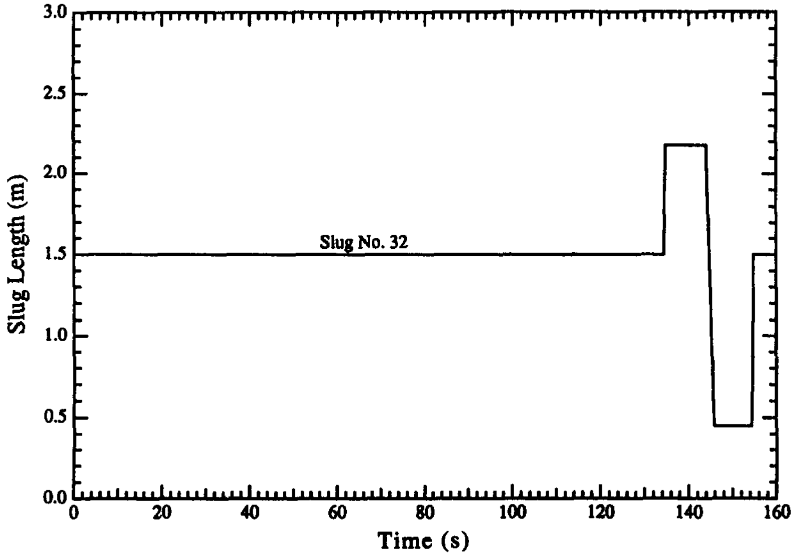



Figure 8. Slug length as a function of time.

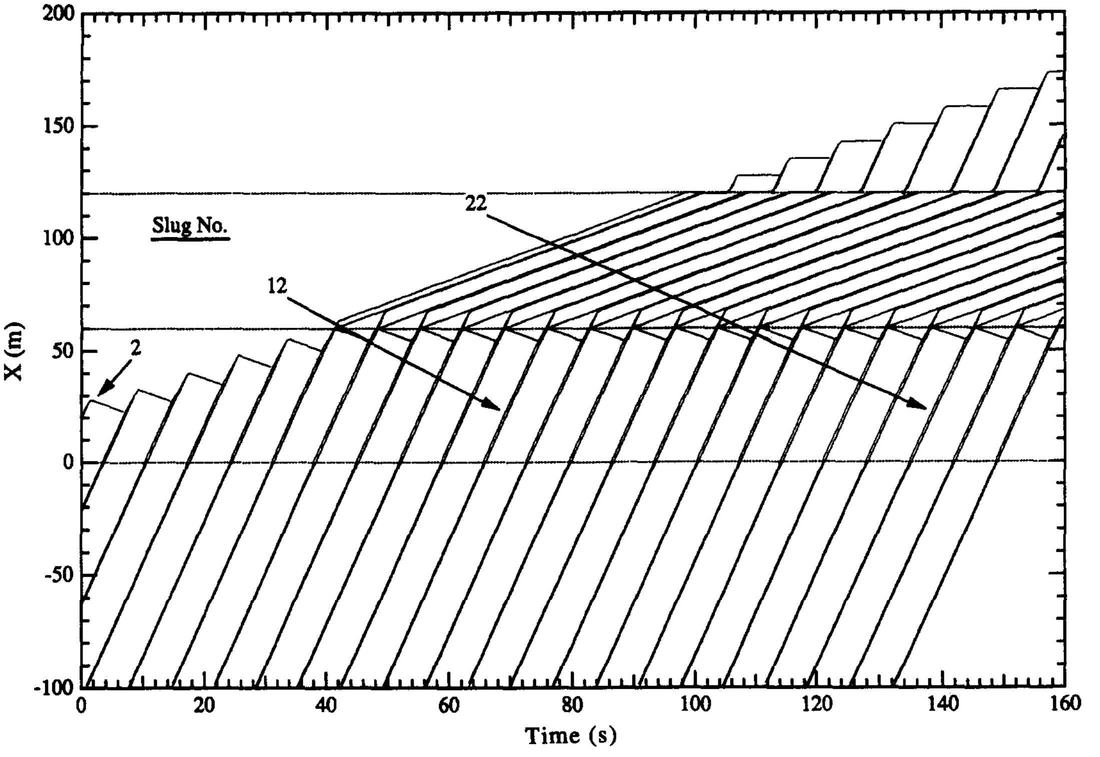



Figure 9. Slug tracking in hilly terrain geometry with slug disappearance downhill.

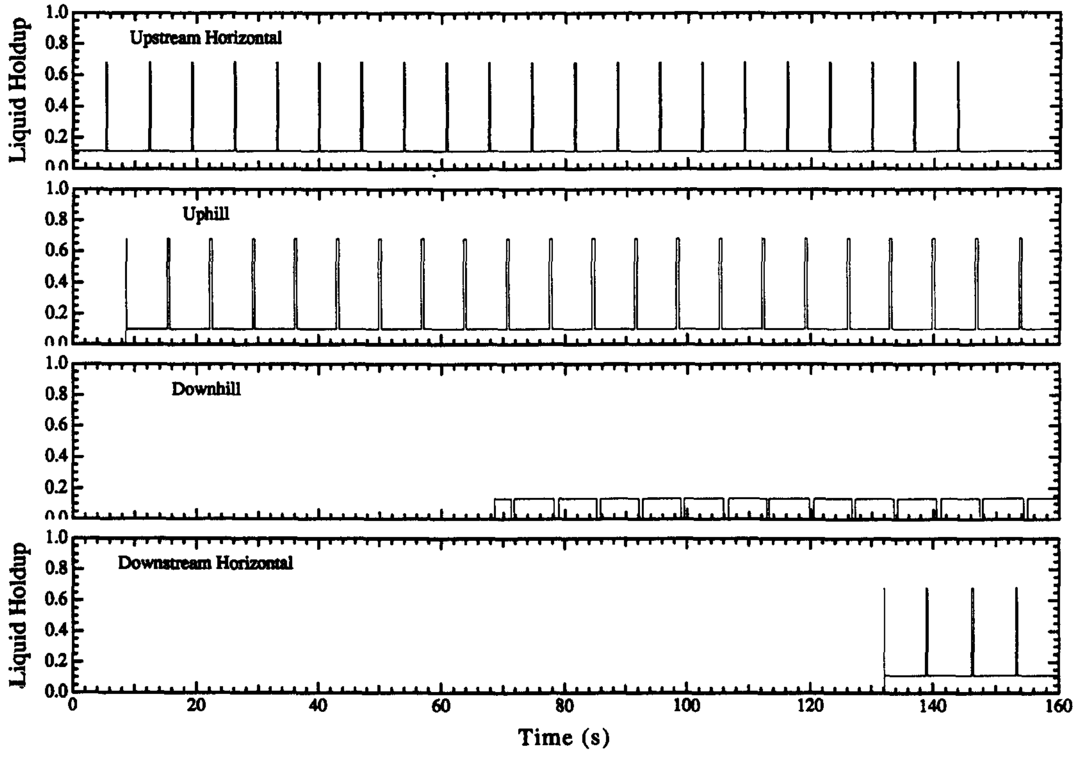



Figure 10. Liquid holdup as a function of time for case 2.

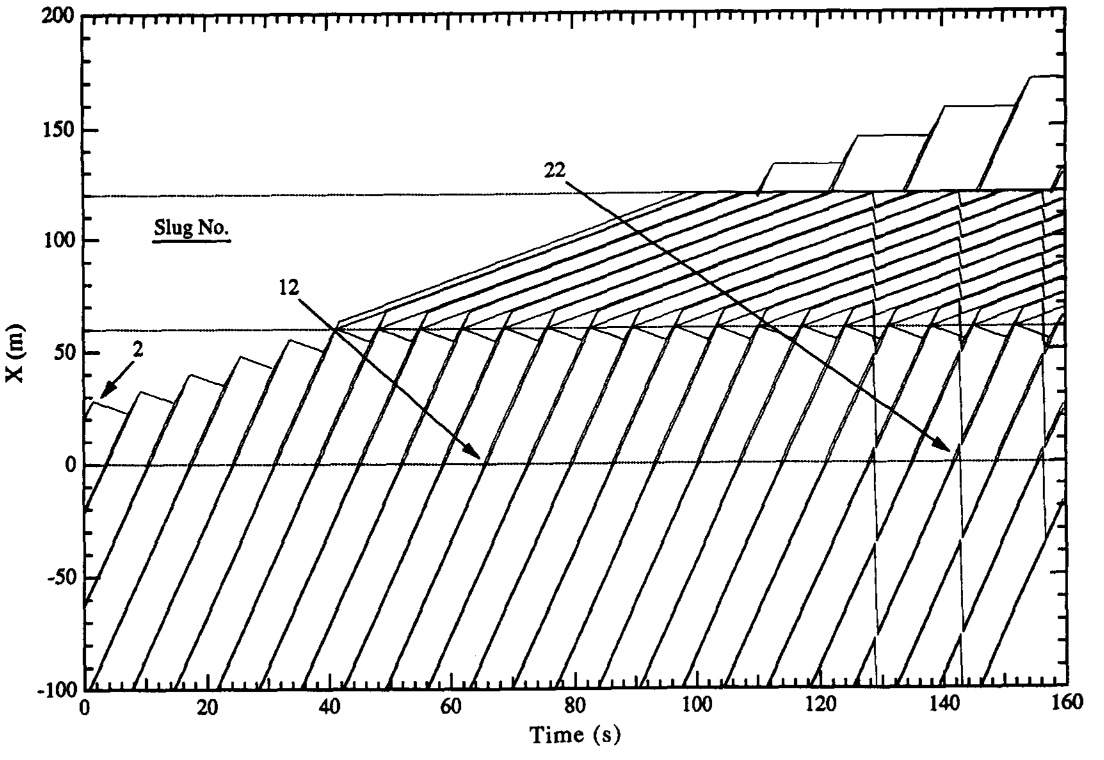



Figure 11. Slug tracking in hilly terrain geometry with slug generation length $l_{gen} = 50D$.

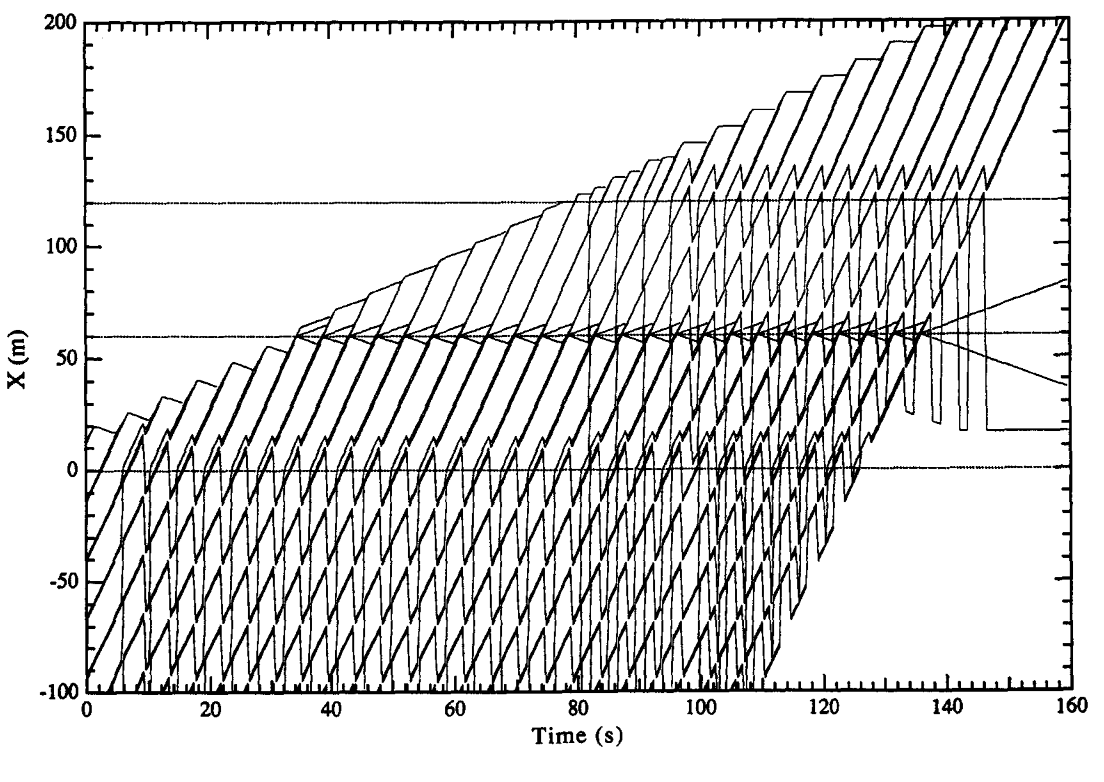



Figure 12. Slug tracking in hilly terrain geometry with slug generation and disappearance.

downhill section the slugs dissipate completely, resulting in a series of films separated from each other and moving downward towards the bottom elbow. This is clearly seen in figure 10 where, in the downhill section (the third curve), the liquid holdup at its maximum is the liquid holdup of the film and is zero in the space between the film boundaries. The liquid flowing downhill towards the bottom elbow accumulates in the elbow and slugs are regenerated at the prescribed generation length $l_{gen}$. Since $l_{gen}$ was taken as $30D$, it is the same as the original slug length in the upstream horizontal section. The result is that the slug length and frequency are recovered exactly in the horizontal downstream section.

A somewhat different situation is demonstrated in figure 11. In this case the generation slug length is $50D$ (2.5 m). Here there is no relation between the slug numbers upstream and those in the horizontal section downstream. As seen, the slug frequency decreases and the slug length as well as the slug unit length (slug + film length) are longer than in the upstream horizontal section (the vertical lines shown in the figure are due to a change in the numbering of the slugs as a result of the disappearance of some slugs).

Figure 12 includes the complex situation when one has slugs generation and disappearance. In this case the slug generation length was set to $l_{gen} = 11D$. Thus, slugs are generated at the elbow $X = 0$. However, the generated slugs length are shorter than the slug stability length ($l_{stab} = 15D$). Therefore the short slugs disappear within a short distance. Figure 13 is a close-up of this process for the first 20 s. As seen, new slugs are now generated between any two slugs. These slugs decrease in length and disappear, owing to the fact that the translational velocity of the slug tail is high for short slugs [12]. Note that a long slug, following a short one generated just ahead of it, increases in length when the short dissipating slug is ahead of it. Otherwise, the slug length remains constant. This is because the liquid lost from the short slug is picked up by the slug behind it. Also, note that when a slug is generated, the length increase of the "normal" slugs entering the uphill section behind it is much less compared to the case without slug generation. This is because the generated new slug has already consumed most of the liquid in the elbow, which otherwise would have been added to the "normal" slug. Figure 12 shows that the process of slug generation and disappearance is repeated in the next bottom elbow ($X = X_{BOTTOM}$). The final configuration in the horizontal

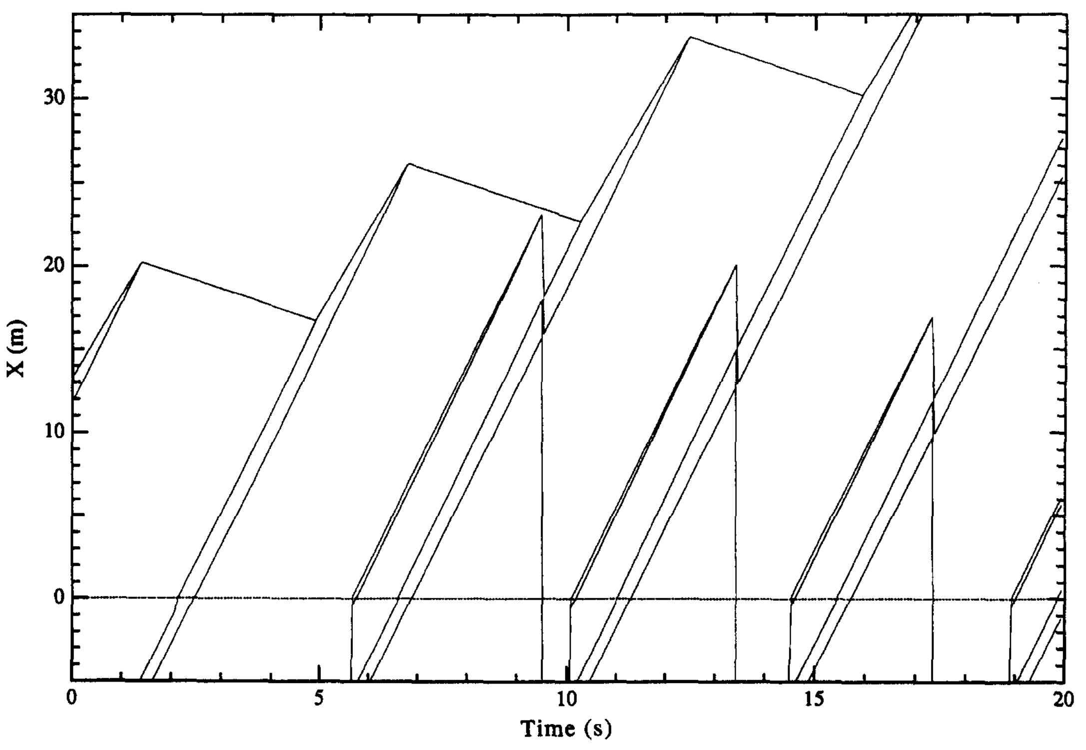



Figure 13. Close-up of figure 12 for the first 20 s.

section is, however, the same as in the original upstream horizontal pipe. That is, the generation and disappearance of the slugs do not, in this case, cause a different flow far downstream. In fact, the same flow configuration downstream is regained as for the simpler case, shown in figure 3, with no generation and disappearance.

## COMPARISON WITH THE EXPERIMENTS

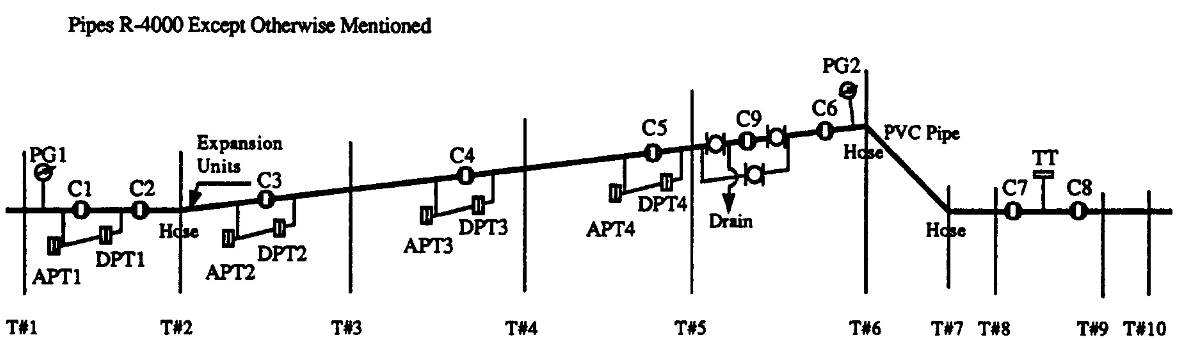
Zheng (1991) and Zheng *et al.* (1992) conducted a detailed experimental study on the behavior of slug flow in a hilly terrain pipeline. They used an air–kerosene two-phase flow loop with a pipeline diameter of 7.79 cm i.d. and a total length of 420.3 m. The pipeline comprised a 203.9 m long upstream horizontal section, a 31.1 m long main inclined section capable of having an inclination angle of $-1^\circ$ to $+5^\circ$ from the horizontal, a 3.7 m long inclined outlet section and a 181.7 m long downstream horizontal section.

Figure 14 shows the instrumentation details for the test facility. Besides a temperature transducer and 4 sets of absolute and differential pressure transducers, there were 9 ring capacitance sensors (C1–C9) located along the pipeline. These ring capacitance sensors allowed determination of instantaneous liquid holdups, slug translational velocities and different slug zone lengths for slug flow in both horizontal and inclined pipes. For the later part of the tests (run numbers $>36$), capacitance sensors C7 and C8 were moved farther downstream to locations 19.5 and 18.0 m before the separator.

The data acquisition system was based on an IBM PC-AT computer. Except for flow rate measurements, which were obtained with orifice and turbine meters, signals from the pressure transducers and capacitance sensors were fed into the computer through an A/D converter.

A total of 115 tests were conducted, resulting in a large amount of high-quality data for slug flow in a hilly terrain pipeline. These tests covered inclination angles of $-1^\circ$, $0^\circ$, $0.5^\circ$, $1^\circ$ $2^\circ$ and $5^\circ$ for the main inclined pipe. Each test contains 50–300 slugs, with a sampling rate of 50 samples/s/channel or higher, depending on the flow rates.

Figures 15–18 contain the simulation runs for 2 cases reported in Zheng (1991), with comparison to the experimental data.




SYMBOLS

APT Absolute Pressure Transducer
DPT Differential Pressure Transducer
PG Pressure Gauge
C Capacitance Sensor
TT Temperature Transducer
Quick Closing/Open Ball Valve

Sensor Locations from Tee # 2 (X = 0 m)

| APT1 | DPT1 | APT2 | DPT2 | APT3 | DPT3 | APT4 | DPT4 | TT |
|---|---|---|---|---|---|---|---|---|
| -3.51 | -2.29 | 2.77 | 3.99 | 12.1 | 13.4 | 21.6 | 22.8 | 37.1 |
| C1 | C2 | C3 | C4 | C5 | C9 | C6 | C7 | C8 |
| -3.66 | -2.13 | 3.11 | 12.6 | 21.4 | 26.2 | 28.3 | 36.5 | 38.0 |
| | | | | | | | 196.9 | 198.5 |

Figure 14. Experimental facility and sensor locations.

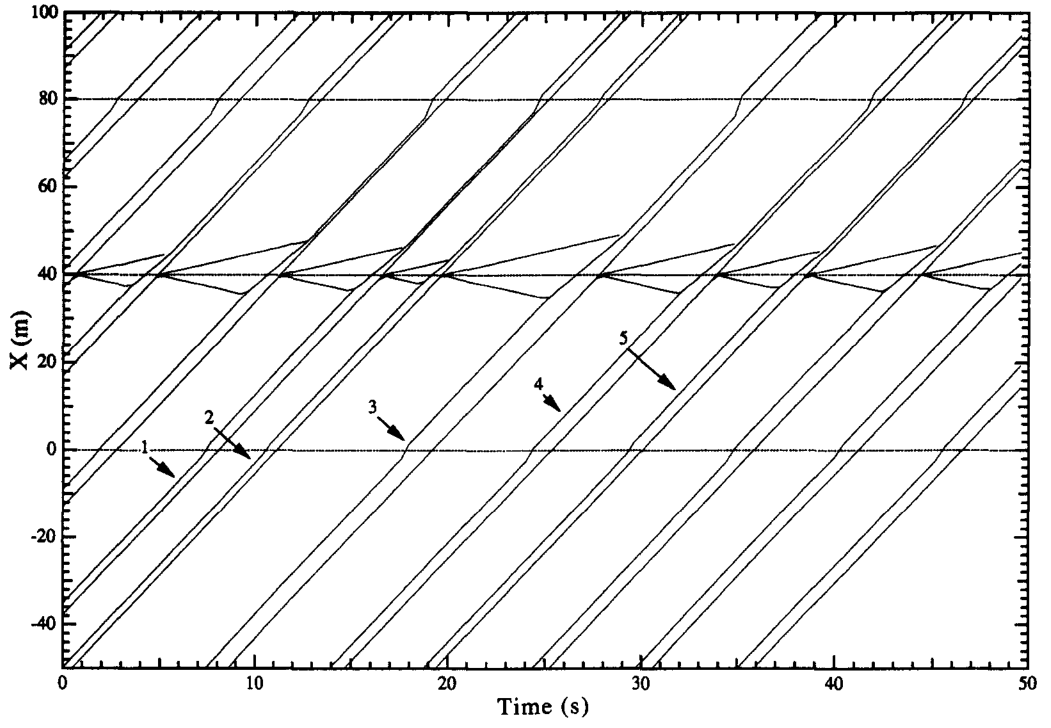



Figure 15. Slug tracking for run 82 ($\beta = 2°$, $U_{LS} = 0.61$ m/s and $U_{GS} = 3.05$ m/s).

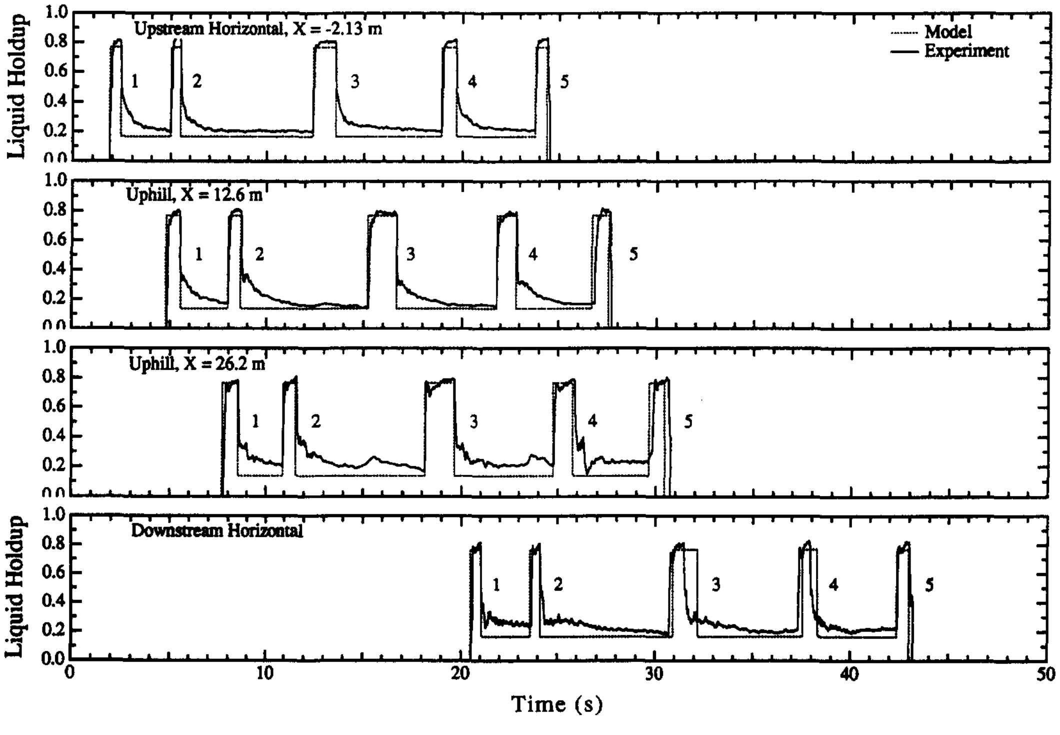



Figure 16. Comparison of the model with the experimental data for run 82 ($\beta = 2°$, $U_{LS} = 0.61$ m/s and $U_{GS} = 3.05$ m/s).

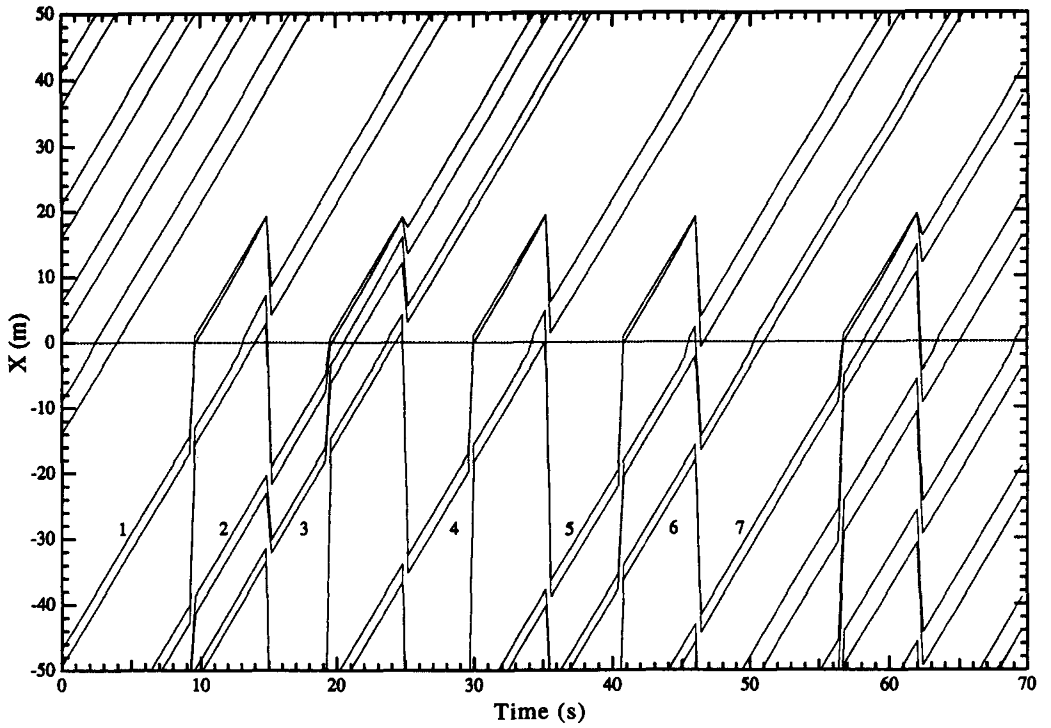



Figure 17. Slug tracking for run 88 ($\beta = 2°$, $U_{LS} = 0.31$ m/s and $U_{GS} = 2.44$ m/s).

The first case shown in figure 15 is a run where no slug generation was observed in the experiment. Only 5 typical slugs were simulated (Nos 1–5). The other slugs shown in the figure were artificially introduced and are not part of the comparison. The original experimental results as measured at 4 locations along the pipe are shown in figure 16 by the solid lines. The top curve is in the horizontal section at $X = -2.13$ m; the second curve is in the uphill section at $X = 12.6$ m; the third curve is also in the uphill section at $X = 26.2$ m; and the bottom curve is far downstream in the horizontal section. For these experimental data the ratio $V_t/U_s = C$ [11] equals 1.28 (rather than 1.2 as assumed before). Once the translational velocity is known, one can input the initial condition, namely the lengths and positions of these 5 slugs in the upstream horizontal section. In the program we also used dummy slugs ahead and behind the 5 slugs to simulate normal slug flow conditions for these 5 slugs, namely to have a liquid film ahead of the first slug. The liquid holdup predicted by the theory is shown in the figure by the dotted line. As can be seen, the match is quite good. In the uphill section it is clearly seen that the slugs become longer and the agreement between the theory and experiment is very good. Obviously, the theoretical results are highly idealized and do not follow the exact chaotic nature of real slug flow. Nevertheless, the comparison does demonstrate the capability of this approach to simulate true slug tracking. The liquid holdup in the liquid slugs is calculated by [13] and is slightly lower in the horizontal case. The holdup of the liquid film falls below the experimental results. However, even these results are quite good if we realize that an equilibrium level is considered in the model, namely the level the liquid film attains far upstream at the point where the shear stresses are in balance with gravity. In practice, the liquid level (holdup) in the film decreases gradually to the equilibrium level far away from the back of the slug (front of the elongated bubble). Part of the discrepancy for the liquid holdup in the film could also be attributed to the inaccuracy of the holdup measurements.

In the slug tracking shown by figure 15, we again see the elongation of the slugs as they pass through the low elbow. No experimental data were taken in the downward inclination section. Nevertheless, in figure 15 a fictitious top elbow was placed at $X = 40$ m and a fictitious low elbow was placed at $X = 80$ m. It is again seen that the slug lengths decrease in the downhill section and regain their original lengths once they enter the horizontal section downstream.

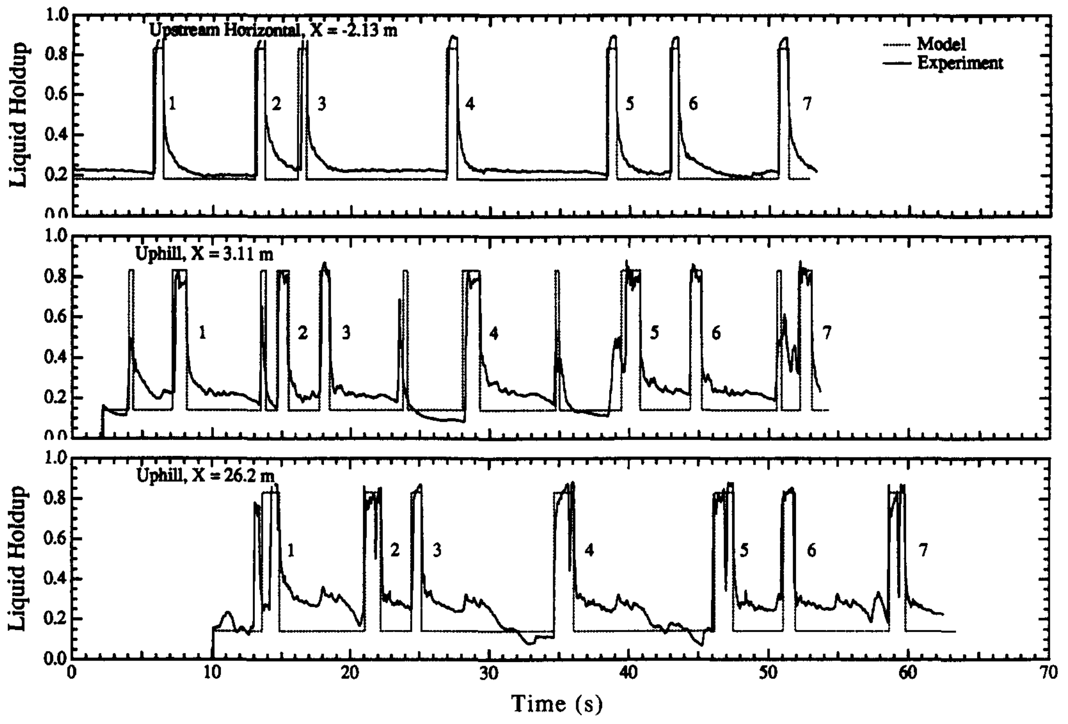



Figure 18. Comparison of the model with the experimental data for run 88 ($\beta = 2°$, $U_{LS} = 0.31$ m/s and $U_{GS} = 2.44$ m/s).

The more complex case with slug generation and disappearance is shown in figures 17 and 18. In this case, 7 experimental slugs initially in the horizontal section are tracked. As can be seen, slugs are generated in the low elbow ahead of slug No. 1 and between slugs Nos 1 and 2, 3 and 4, 4 and 5, and 6 and 7. For this simulation we assigned $l_{gen} = 15D$, $l_{stab} = 16D$, $C = 1.28$ and $U_{bmax} = 1.2CU_s$ [12]. As seen, all generating slugs disappear within a short distance. A comparison between the simulated and experimental results is shown in figure 18. The middle curve shows the liquid holdup at a location quite close to the low elbow ($X = 3.11$ m). The locations and the lengths of the slugs in the uphill section can be simulated quite well by the theory. The mechanisms of slug generation, disappearance and growth are well-demonstrated in this case. For example, slug No. 2 almost does not elongate in the low elbow because the liquid accumulated in the low elbow was "used" to generate the new slug ahead of slug No. 2. Since the slug generated is short, its length decreases as it travels downstream and the length of slug No. 2 increases as it picks up the liquid shed by this short unstable slug. This process can be clearly observed in figure 17. Once the short slug disappears, the slug length remains constant. In figure 18 one can also observe that slug No. 2 in the first uphill location (second curve) is almost the same in length as in the upstream horizontal section. However, once the generated slug disappears, the liquid in this slug is added to slug No. 2, which increases in length as shown in the bottom curve for the $X = 26.2$ m location.

Admittedly, the comparison with experiment is quite limited. The purpose in this work is just to demonstrate the applicability of the proposed model to simulate the main features of slug behavior in a hilly terrain pipeline.

## SUMMARY AND CONCLUSIONS

A model is presented that is capable of slug tracking and simulating slug behavior in elbows where a change of inclination occurs. Comparison with data demonstrates its applicability. The model simulates the change in slug length for both top and low elbows. Also, the model adequately describes slug generation in low elbows, the dissipation of unstable short slugs behind long ones, as well as the possible dissipation of slugs at top elbows.

The model requires as an input the superficial mixture velocity ($U_s = U_{LS} + U_{GS}$), the functional dependence of the translational velocity, $V_t$, on the superficial velocity and the liquid slug length ahead of the elongated bubble (note that [12] is just an example used here to demonstrate capability and not necessarily a correct form), the liquid slug holdup relation [13], the lengths of the generated slugs, $l_{gen}$, the stability length, $l_{stab}$, and the equilibrium film velocity as a function of its holdup (in this work the film velocity was considered constant). These input variables can be obtained either via appropriate correlations or from detailed models that can predict the detailed local behavior. In addition, the initial condition of all slugs present in the pipe, or generated at the entrance, should be input.

Admittedly, the model presented here is very approximate. It considers films of constant thickness and does not take into account the change in the film thickness with the relative distance from the back of the slug. Nevertheless, the main physical behavior of slugs in hilly terrain pipelines is well-modeled.

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